代数幾何学シンポジウム記録
1982年度 pp.132-133

Constrution of algebraic vector bundles of rank 2 on non-singular algebraic varieties of arbitrary dimensions

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As for construction of algebraic vector bundles on non-singular algebraic varieties, the following methods are wellknown.

(1) J.P.Serre : 2-bundles associated to closed subschemes of codimension 2, locally complete intersection.

(2) R.Schwarzenberger :

(a) Ramified 2-coverings.

(b) Blowing-ups + Descents.

(3) G.Horrocks : Monads.

(4) M.Maruyama : Elementary transformations.

Using the methods (2), R.Schwarzenberger has constructed many indecomposable 2-bundles on algebraic surfaces. In this note, we shall report that we can generalize the method (2) (b) to algebraic varieties of arbitrary dimensions and obtain the following.

Theorem. Let X be a non-singular algebraic variety defined over an algebraically closed field k (char $k = p \geq 0$) and let $\{Y, Z\}$ be a pair of closed subschemes of X satisfying the following conditions :

(a) D is a reduced divisor of X whose singular locus $\mathrm{Sing}(D)$ is either empty or of $\mathrm{codim}_X \mathrm{Sing}(D) = 4$.

(b) Z is a smooth closed subscheme of D with $\mathrm{codim}_X Z = 2$ and it contains $\mathrm{Sing}(D)$.

(c) There is a rational map $f : D \longrightarrow P^1$ such that the regular domain $D(f)$ of f contains $D - \mathrm{Sing}(D)$ and $Z = f^{-1}(0)$ scheme theoretically.

Then there are an algebraic 2-bundle E on X and a section s of E satisfying the following properties :

(1) Z coincides with $Z(s)$, the scheme of zeros of s.

(2) $O_X(D)$ is isomorphic to $\Lambda^2 E$.

Moreover,

(3) if $H^1(X, O_X) = 0$, then there exists another section t of E

such that D coincides with $Z(s\wedge t)$, the scheme of zeros of $s\wedge t$.

(4) if X is projective and Z is connected and if $H^1(X,O_X(-D)) = 0$, then E is determined uniquely up to isomorphisms.

As a corollary, we obtain the following.

Corollary. Let X be a non-singular projective variety defined over an algebraically closed field k (char $k = p \geq 0$), Y a non-singular divisor of X and let $\{Z_t\}_{t\in P^1}$ be a Lefshetz pencil on Y with base locus W. Then there exist a reflexive sheaf E of rank 2 on X and a section s of E such that there is an exact sequence

$$0 \longrightarrow O_X \longrightarrow E \longrightarrow O_X(Y)\otimes I_{Z_t} \longrightarrow 0$$

,where I_{Z_t} is the defining ideal sheaf of Z_t in X and Sing(E) coincides with W.

The proof of the above theorem and corollary wil be published elsewhere.